

Consider two lightcurves  $x(t)$  and  $y(t)$ , where  $x(t)$  is the driving lightcurve and  $y(t)$  is the reprocessed lightcurve. If they are related by a linear impulse response,  $g(\tau)$ , then:

$$y(t) = \int_{-\infty}^{\infty} g(\tau)x(t - \tau)d\tau \quad (1)$$

So,  $y(t)$  is a delayed and blurred version of  $x(t)$ , with the amount of delay and blurring encoded in  $g(\tau)$ .

The power spectral density (PSD) of  $x(t)$  is calculated from the Fourier transform of  $x(t)$ , which we denote  $X(\nu)$ . The PSD is  $|X(\nu)|^2 = X^*(\nu)X(\nu)$ , where the  $*$  denotes the complex conjugate. From the convolution theorem of Fourier transforms we can write:

$$Y(\nu) = G(\nu)X(\nu) \quad (2)$$

This means it is easy to relate the PSD of the reprocessed lightcurve to the PSD of the driving lightcurve and the impulse response function:

$$|Y(\nu)|^2 = |G(\nu)|^2|X(\nu)|^2 \quad (3)$$

The cross spectrum is defined as

$$C(\nu) = X^*(\nu)Y(\nu) \quad (4)$$

the phase,  $\phi$ , of which gives the phase lag between X and Y at each Fourier frequency,  $\nu$ . This can be converted to a time lag through:

$$\tau(\nu) = \frac{\phi(\nu)}{2\pi\nu} \quad (5)$$

Since  $Y(\nu) = G(\nu)X(\nu)$ , the cross spectrum can be written as:

$$C(\nu) = X^*(\nu)G(\nu)X(\nu) = G(\nu)|X(\nu)|^2 \quad (6)$$

thus, for a given impulse response function, one can trivially predict the time lags as a function of frequency,  $\tau(\nu)$ , by calculating the phase of  $G(\nu)$ , and the frequency dependence of the lags directly relates to the shape of the response function.