Problem 1

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a) The wave function $\psi(\xi) = \xi^p \exp(-\xi^2/2)\ L(\xi^2)$, should satisfy the boundary condition for either even or odd fuctions of ξ with p=0 or p=1, respectively, for any function L(u).

Recall $\xi = \sqrt{(m\omega/\hbar)}\ x$.

$d\xi = \sqrt{(m\omega/\hbar)}\ dx = \alpha\ dx$.

$\psi(\xi) = \xi^p \exp(-\xi^2/2)\ L(\xi^2)$.

$$\begin{aligned}\psi'(\xi) &= d/d\xi\ (\xi^p \exp(-\xi^2/2)\ L(\xi^2)) \\ &= (d/d\xi\ \xi^p) \exp(-\xi^2/2)\ L(\xi^2) \\ &\quad + \xi^p\ (d/d\xi \exp(-\xi^2/2))\ L(\xi^2) \\ &\quad + \xi^p \exp(-\xi^2/2)\ (d/d\xi\ L(\xi^2)).\end{aligned}$$

For an even state, $\psi'(0) = 0$ and p=0, so

$$\begin{aligned}\psi'(\xi) &= (d/d\xi\ 1) \exp(-\xi^2/2)\ L(\xi^2) \\ &\quad + 1\ d/d\xi \exp(-\xi^2/2)\ L(\xi^2) \\ &\quad + 1 \exp(-\xi^2/2)\ d/d\xi\ L(\xi^2).\end{aligned}$$

$$\begin{aligned}\psi'(\xi) &= -\ \xi \exp(-\xi^2/2)\ L(\xi^2) \\ &\quad + \exp(-\xi^2/2)\ L'(\xi^2).\end{aligned}$$

$\psi'(0) = L'(0)$.

This must be wrong, somehow... in this case the boundary condition only applies if L'(0) = 0.

■

For an odd state, $\psi(0) = 0$ and p=1, so

$\psi(0) = 0^p * \ldots = 0$. This is trivial.

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b) Equation 2.72 from Griffiths:

$dH_n/d\xi = 2nH_{n-1}(\xi)$.

Substituting the state $\psi(\xi)$, I can obtain the associated Laguerre differential equation

$uL'' + (p - \frac{1}{2} + 1 - u)L' + kL = 0$, with $u = \xi^2$.

A better and equivalent substitution uses
$\psi(x,t) = A \exp(\imath(kx - (\hbar k^2/2m)t))$.

$h = \xi^p L(\xi^2)$.

$u = \xi^2$.

$\xi^p = \xi^2$

$h = \xi^p L(\xi^2)$

$\psi(x,t) = A \exp(\imath(kx - k^2/4\pi m\ \xi^p L(\xi^2)\ t))$
$= A \exp(\imath(kx - k^2/4\pi m\ \xi^p L(u)\ t))$.

This... is probably not the "h" you meant.

$h = \xi^p L(\xi^2)$.

$dH_n/d\xi = 2nH_{n-1}(\xi)$.

$h'(\xi) = dh/d\xi = p\xi^{p-1} L(\xi^2) + \xi^p L'(\xi^2)$.

$h''(\xi) = p\xi^{p-1} L(\xi^2) + \xi^p L'(\xi^2) = p(p-1)\xi^{p-2} L(\xi^2) + p\xi^{p-1} L'(\xi^2) + p\xi^{p-1} L'(\xi^2) + \xi^p L''(\xi^2)$.

$p(p-1)\xi^{p-2} L(\xi^2) + p\xi^{p-1} L'(\xi^2) + p\xi^{p-1} L'(\xi^2) + \xi^p L''(\xi^2)$.

H_n in Griffiths maps to h_p in the problem sheet, I'll assume.

$H_n = \xi^n$.