A particle in an infinite square well has an initial state vector, with A a real number and ι the imaginary unit,

$$|\Psi(t=0)\rangle = A(|\psi_1\rangle - |\psi_2\rangle + \iota|\psi_3\rangle).$$

where $|\psi_n\rangle$ are the energy eigenstates. This also means

$$\langle\Psi(t=0)| = A(\langle\psi_1| - \langle\psi_2| - \iota\langle\psi_3|)$$

In the energy basis,

$$|\psi_1\rangle \doteq \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad |\psi_2\rangle \doteq \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \text{and } |\psi_3\rangle \doteq \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

So,

$$|\Psi(t=0)\rangle \doteq \begin{pmatrix}A\\-A\\\iota A\end{pmatrix}.$$

(**a**) The state vector is normalized by taking the quotient of the magnitude.

$$|\Psi'(t=0)\rangle \doteq \frac{A}{\sqrt{(3A^2)}}\begin{pmatrix}1\\-1\\\iota\end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\\iota\end{pmatrix}.$$

When the Hamiltonian operates on $|\psi_n\rangle$ with results according to the general eigenvalue equation, with E_n the measured energy of state n,

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle.$$

The measured energies for a point particle in an infinite square well are given by, with L the x-width of the well, and m the particle mass,

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}.$$

So,

$$\hat{H}|\Psi'(t=0)\rangle = \frac{1}{\sqrt{3}}(E_1|\psi_1\rangle - E_2|\psi_2\rangle + \iota E_3|\psi_3\rangle).$$

(**b**) There is an equal chance of measuring any of the three values, so $\mathcal{P}_n$=1/3. The measured enemies are given by the previous expression.

Energy	Probability
$\frac{\pi^2\hbar^2}{2mL^2}$.	$\mathcal{P}_1 = 1/3$
$\frac{2\pi^2\hbar^2}{mL^2}$.	$\mathcal{P}_2 = 1/3$
$\frac{9\pi^2\hbar^2}{2mL^2}$.	$\mathcal{P}_3 = 1/3$

The average value of the energy, or the expectation value, is given by

$$\left|\langle\Psi'|\hat{H}|\Psi'\rangle\right|^2 = \left|\frac{1}{\sqrt{3}}(\langle\psi_1| - \langle\psi_2| - \iota\langle\psi_3|)\frac{1}{\sqrt{3}}(E_1|\psi_1\rangle - E_2|\psi_2\rangle + \iota E_3|\psi_3\rangle)\right|^2.$$

$$\langle\Psi'|\hat{H}|\Psi'\rangle = \frac{1}{\sqrt{3}}(\langle\psi_1| - \langle\psi_2| - \iota\langle\psi_3|)\frac{1}{\sqrt{3}}(E_1|\psi_1\rangle - E_2|\psi_2\rangle + \iota E_3|\psi_3\rangle)$$

$$= 1/3\ (\langle\psi_1| - \langle\psi_2| - \iota\langle\psi_3|)(E_1|\psi_1\rangle - E_2|\psi_2\rangle + \iota E_3|\psi_3\rangle)$$

$$= 1/3\ (\langle\psi_1|E_1|\psi_1\rangle - \langle\psi_1|E_2|\psi_2\rangle + \langle\psi_1|\iota E_3|\psi_3\rangle)\ (-\langle\psi_2|E_1|\psi_1\rangle + \langle\psi_2|E_2|\psi_2\rangle - \langle\psi_2|\iota E_3|\psi_3\rangle)$$

$$(-\imath\langle\psi_3|E_1|\psi_1\rangle + \imath\langle\psi_3|E_2|\psi_2\rangle - \imath\langle\psi_3|\imath E_3|\psi_3\rangle).$$

Orthogonality is a property of the energy eigenstates:

$$\langle E_n|E_m\rangle = \delta_{nm}, \text{ where } \delta_{nm} = \begin{cases} 0 \text{ if } n\neq m \\ 1 \text{ if } n=m \end{cases}, \text{ so}$$

$$\langle\Psi'|\hat{H}|\Psi'\rangle = {}^1/_3\ (\langle\psi_1|E_1|\psi_1\rangle)\ (\langle\psi_2|E_2|\psi_2\rangle)\ (-\imath\langle\psi_3|\imath E_3|\psi_3\rangle)$$

$$= {}^1/_3\ E_1\ E_2\ E_3 = {}^1/_3\ \frac{\pi^2\hbar^2}{2mL^2}\ \frac{4\pi^2\hbar^2}{2mL^2}\ \frac{9\pi^2\hbar^2}{2mL^2} = \frac{3}{2}\left(\frac{\pi^2\hbar^2}{mL^2}\right)^3.$$

(**c**) Therefore, the energy expectation value

$$|\langle\Psi'|\hat{H}|\Psi'\rangle|^2 = \frac{9}{4}\left(\frac{\pi^2\hbar^2}{mL^2}\right)^6.$$

Because the hamiltonian is time-independent, the state vector progresses timewise according to,

$$|\Psi'(t)\rangle = \exp(-\imath\frac{\hat{H}t}{\hbar})|\Psi'(t=0)\rangle = \frac{1}{\sqrt{3}}\exp(-\imath\frac{\hat{H}t}{\hbar})\ (|\psi_1\rangle - |\psi_2\rangle + \imath\ |\psi_3\rangle).$$

$$\frac{1}{\sqrt{3}}\left(\exp(-\imath\frac{E_1t}{\hbar})|\psi_1\rangle - \exp(-\imath\frac{E_2t}{\hbar})|\psi_2\rangle + \imath\ \exp(-\imath\frac{E_3t}{\hbar})|\psi_3\rangle\right)$$

(**d**)

$$= \frac{1}{\sqrt{3}}\left(\exp(-\imath\ \frac{\pi^2\hbar t}{2mL^2})|\psi_1\rangle - \exp(-\imath\ \frac{2\pi^2\hbar t}{mL^2})|\psi_2\rangle + \imath\ \exp(-\imath\ \frac{9\pi^2\hbar t}{2L^2})|\psi_3\rangle\right).$$

What are the possible measurements at time $t = \hbar/E_1$?

(**e**) The same value of energy will be measured for each state. Since there has been no change to the coefficients besides a change in phase, and the phase term goes to 1 under the modulus, the probabilities remain the same.

$$|\Psi'(t=\hbar/E_1)\rangle = \frac{1}{\sqrt{3}}\left(\exp(-\imath)|\psi_1\rangle - \exp(-\imath\ \frac{E_2}{E_1})|\psi_2\rangle + \imath\ \exp(-\imath\ \frac{E_3}{E_1})|\psi_3\rangle\right)$$

Energy	Probability
$\frac{\pi^2\hbar^2}{2mL^2}$.	$\mathcal{P}_1 = {}^1/_3$
$\frac{2\pi^2\hbar^2}{mL^2}$.	$\mathcal{P}_2 = {}^1/_3$
$\frac{9\pi^2\hbar^2}{2mL^2}$.	$\mathcal{P}_3 = {}^1/_3$