7.35 is nothing more than a definition of spherical coordinates.

$$\begin{cases} x = r \sin\theta \cos\phi \\ y = r \sin\theta \sin\phi \\ z = r \cos\theta \end{cases}$$

7.47 is the set of algebraic conditions expressed by the vector definition $\mathbf{L} = \mathbf{r} \times \mathbf{p}$.

$$\begin{cases} \hat{L}_x = y p_z - z p_y = -\iota\hbar \left(y \frac{\partial}{\partial y} - z \frac{\partial}{\partial y}\right) \\ \hat{L}_y = z p_x - x p_z = -\iota\hbar \left(z \frac{\partial}{\partial y} - x \frac{\partial}{\partial y}\right) \\ \hat{L}_z = x p_y - y p_x = -\iota\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial y}\right) \end{cases}$$

Substituing 7.35 into 7.47,

$$\begin{cases} \hat{L}_x = r \sin\theta \sin\phi \, p_z - r \cos\theta \, p_y = -\iota\hbar \left(r \sin\theta \sin\phi \frac{\partial}{\partial y} - r \cos\theta \frac{\partial}{\partial y}\right) \\ \hat{L}_y = r \cos\theta \, p_x - r \sin\theta \cos\phi \, p_z = -\iota\hbar \left(r \cos\theta \frac{\partial}{\partial y} - r \sin\theta \cos\phi \frac{\partial}{\partial y}\right) \\ \hat{L}_z = r \sin\theta \cos\phi \, p_y - r \sin\theta \sin\phi \, p_x = -\iota\hbar \left(r \sin\theta \cos\phi \frac{\partial}{\partial y} - r \sin\theta \sin\phi \frac{\partial}{\partial y}\right) \end{cases}$$