A spin-1/2 particle with a magnetic moment is known to be in the state $|\Psi(t=0)\rangle = |+\rangle$.

a) If the observable S_x is measured at t=0, the possible results are $\hbar/2$ and $-\hbar/2$ with an equal probability of measuring either.

b) The system evolves in a uniform magnetic field $\mathbf{B} = B_0\hat{y}$. What is the state of the system at t=T?

Since the magnetic field is oriented along the y axis, the energy eigenstates will be associated with that direction. The eigenstates are $|\pm_y\rangle$.

The Hamiltonian for this system is

$$H = -\omega_0 S_y, \text{ with } \omega_0 = g\, q/2m_e\, B_0 \approx e/m_e\, B_0.$$

The eigenvalue equations in the energy basis are therefore

$$\hat{H}|E\pm\rangle = \omega_0 \pm\hbar/2\, |E\pm\rangle = \omega_0 \pm\hbar/2\, |\pm\rangle = E\pm\, |\pm\rangle$$

The initial state is prepared to $|+\rangle$, which means, in the y basis,

$$|\Psi(t=0)\rangle = 1/\sqrt{2}\, |+_y\rangle - 1/\sqrt{2}\, |-_y\rangle$$

This Hamiltonian is time independent, so the time evolution is given by multiplying each eigenstate with the time-pdependent phase factor, with $\imath$ the imaginary unit.

$$|\Psi(t)\rangle = \exp(-\imath\, E_+\, t/\hbar)/\sqrt{2}\, |+_y\rangle - \exp(-\imath\, E_-\, t/\hbar)/\sqrt{2}\, |-_y\rangle$$

If the time-evolution of the general state is known in the energy basis (aligned with the spin y basis), the time-evolved state at some time t in the z basis can be predicted by projecting the z basis onto the state to determine the coefficients in the z basis. In mathematical terms,

$$\langle +|\Psi(t)\rangle|+\rangle =$$
$$\langle -|\Psi(t)\rangle|-\rangle = ($$