$\hat{x}|x\rangle = x|x\rangle$

$|x\rangle$ forms a continuous state

$$|\Psi\rangle = \sum_i c_i |x\rangle \rightarrow \int_{-\infty}^{\infty} dx \ \langle x|\Psi\rangle |x\rangle$$

We have

$$1 = \int_{-\infty}^{\infty} dx \ |x\rangle\langle x|$$

$$|\Psi\rangle = \mathbb{1}|\Psi\rangle \rightarrow \int dx \ \langle x|\Psi\rangle |x\rangle$$

In particular,

$$|x'\rangle = \int_{-\infty}^{\infty} dx \ \langle x|x'\rangle |x\rangle = \delta(x-x')$$

Also

$$\langle\Psi|\Psi\rangle = \langle\Psi|\mathbb{1}\,\mathbb{1}|\Psi\rangle$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx \ dx' \ \langle\Psi|x\rangle \ \langle x|x'\rangle \ \langle x'|\Psi\rangle$$

$$= \int_{\infty}^{-\infty} dx \ \langle\Psi|x\rangle \ \langle x|\Psi\rangle$$

$$= \int_{-\infty}^{\infty} dx \ |\langle x|\Psi\rangle|^2 = 1$$