The characteristic equation for the spin operator S_y is

$$S_y (S_y + \hbar)(S_y - \hbar) = 0.$$

The eigenvalues, which are the roots of this equation, are

$$\lambda = 0, \pm\hbar.$$

The S_y operator is already diagonalized in its own basis, so the matrix has the immediately constructable form of

$$S_y \doteq \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

To produce the operator S_y, one can apply the rotation matrix for a rotation about the y axis., where the angle of rotation is $\pi/2$.

$$R = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$S_x = S_y R \doteq \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

The same is true for the S_x operator in its basis. To express this operator in the z basis, however, it must be diagonalized.