Angular momentum system is prepared in the state

$$|\Psi\rangle = \frac{1}{\sqrt{10}}|11\rangle - \frac{2}{\sqrt{10}}|10\rangle + \frac{\imath 2}{\sqrt{10}}|22\rangle + \frac{\imath 1}{\sqrt{10}}|20\rangle$$

Possible results of $\mathbf{L}^2$ measurement?

The $\mathbf{L}^2$ operator was developed in lecture and in the text. The related eigenvalue equation is

$$\mathbf{L}^2|lm\rangle = l(l+1)\,\hbar^2|lm\rangle$$

The possible measurements of $\mathbf{L}^2$ for this system are those associated with the initial state vector, i.e. the values lm = 11, 10, 22, 20.

For lm = 11 & 10, $\mathbf{L}^2 = 2\hbar^2$

For lm = 22 & 20, $\mathbf{L}^2 = 6\hbar^2$.

Since the same $\mathbf{L}^2$ is measures for two states, the sum of the probabilities of measuring those states is the probability of measuring that squared angular momentum.

$\mathcal{P}(\mathbf{L}^2=2\hbar^2) = |\langle 11|\Psi\rangle|^2 + |\langle 10|\Psi\rangle|^2$.

$$\langle 11|\Psi\rangle = \langle 11|\left(\frac{1}{\sqrt{10}}|11\rangle - \frac{2}{\sqrt{10}}|10\rangle + \frac{\imath 2}{\sqrt{10}}|22\rangle + \frac{\imath 1}{\sqrt{10}}|20\rangle\right).$$

The eigenstates for this system are LaPlace's spherical harmonic functions, which comprise an orthogonal set, I.E.:

$$\langle lm|l'm'\rangle = \delta_{ll'}\,\delta_{mm'}.$$

$$|\langle 11|\Psi\rangle|^2 = \left|\langle 11|\frac{1}{\sqrt{10}}|11\rangle\right|^2 = {}^1/_{10}.$$

$$|\langle 10|\Psi\rangle|^2 = \left|\langle 10|\frac{2}{\sqrt{10}}|10\rangle\right|^2 = {}^4/_{10}.$$

(**a**)

$\mathcal{P}(\mathbf{L}^2=2\hbar^2) = ½$.

No need to calculate the other set: since the vector is normalized, the probability of measuring $\mathbf{L}^2=6\hbar^2$ is also

$\mathcal{P}(\mathbf{L}^2=6\hbar^2) = ½$.

For $\mathbf{L}_z$ the eigenvalue equation is

$$\mathbf{L}_z|lm\rangle = m\hbar|lm\rangle,$$

so the expected measurements are,

for lm = 11 & 21, $\mathbf{L}_z = \hbar$.

for lm = 10 & 20, $\mathbf{L}_z = 0$.

In this case, the probabilities will be

$\mathcal{P}(\mathbf{L}_z=\hbar) = |\langle 11|\Psi\rangle|^2 + |\langle 21|\Psi\rangle|^2$ and

$\mathcal{P}(\mathbf{L}_z=0) = |\langle 10|\Psi\rangle|^2 + |\langle 20|\Psi\rangle|^2$.

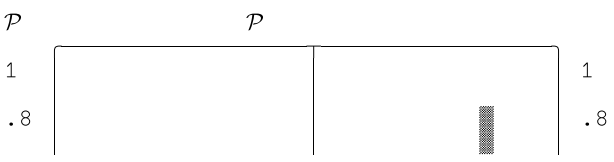
The method has already been demonstrated, so taking the probability components and summing,

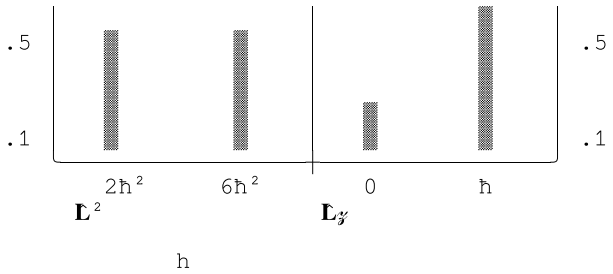
(**b**)

$\mathcal{P}(\mathbf{L}_z=\hbar) = {}^2/_{10}$.

$\mathcal{P}(\mathbf{L}_z=0) = {}^8/_{10}$.

Histogram of probabilities:




.5
.1
.5
.1
2ħ²
6ħ²
0
ħ
L²
L_z
h