There is an angular momentum system with the state function

$$|\Psi\rangle = \frac{2}{\sqrt{29}}|1\ 1\rangle + \imath\frac{3}{\sqrt{29}}|1\ 0\rangle - \frac{4}{\sqrt{29}}|1\ -1\rangle$$

In general the eigenvalue equation for the $\hat{L}_z$ operator is

$\hat{L}_z|l\ m\rangle = m\hbar|l\ m\rangle$, where $m\hbar$ are the possible measurements.

The possible measurements of this system, then, are, for m = {-1, 0, 1}:

$-\hbar$, 0, $\hbar$.

The probability for is given by

$|\langle 1\ m'|\Psi\rangle|^2$, with m' = {-1, 0, 1}.

The eigenstates form an orthogonal set such that

$\langle l'\ m'|l\ m\rangle = \delta_{ll'}\delta_{mm'}$.

Then,

$$\langle 1\ 1|\Psi\rangle = \langle 1\ 1|\left(\frac{2}{\sqrt{29}}|1\ 1\rangle + \imath\frac{3}{\sqrt{29}}|1\ 0\rangle - \frac{4}{\sqrt{29}}|1\ -1\rangle\right)$$

$$= \langle 1\ 1|\frac{2}{\sqrt{29}}|1\ 1\rangle = \frac{2}{\sqrt{29}}.$$

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$$|\langle 1\ 1|\Psi\rangle|^2 = \frac{4}{29} = {}^4/_{29}.$$

Similarly,

$$|\langle 1\ 0|\Psi\rangle|^2 = \frac{9}{29} = {}^9/_{29} \text{ and}$$

$$|\langle 1\ -1|\Psi\rangle|^2 = \frac{16}{29} = {}^{16}/_{29}.$$

The eigenvalue equations for the $\hat{L}_z$ operator are simplified because $\hat{L}_z$ is diagonal in the z basis. The $\hat{L}_x$ operator produces the same measurements, but the matrix representation of the $\hat{L}_x$ operator must be applied. It is

$$\hat{L}_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Applying the operator to the states in Ψ,

$$\hat{L}_x|1\ 1\rangle \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{\hbar}{\sqrt{2}}|1\ 0\rangle.$$

$$\hat{L}_x|1\ 0\rangle \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix} = \frac{\hbar}{\sqrt{2}}(|1\ 1\rangle + |1\ -1\rangle), \text{ and}$$

$$\hat{L}_x|1\ -1\rangle \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{\hbar}{\sqrt{2}}|1\ 0\rangle.$$

$$\hat{L}_x|\Psi\rangle = \left(\frac{2}{\sqrt{29}}\hat{L}_x|1\ 1\rangle + \imath\frac{3}{\sqrt{29}}\hat{L}_x|1\ 0\rangle - \frac{4}{\sqrt{29}}\hat{L}_x|1\ -1\rangle\right)$$

$$\frac{2}{\sqrt{29}}\hat{L}_x|1\ 1\rangle = \frac{2}{\sqrt{58}}\hbar|1\ 0\rangle,$$

$$\imath\frac{3}{\sqrt{29}}\hat{L}_x|1\ 0\rangle = \imath\frac{3}{\sqrt{58}}\hbar(|1\ 1\rangle + |1\ -1\rangle), \text{ and}$$

$$\frac{4}{\sqrt{29}}\hat{L}_x|1\ -1\rangle = \frac{4}{\sqrt{58}}\hbar|1\ 0\rangle.$$

Then,

$$\hat{L}_x|\Psi\rangle = \hbar\left(\frac{-2}{\sqrt{58}}|1\ 0\rangle + \imath\frac{3}{\sqrt{58}}(|1\ 1\rangle + |1\ -1\rangle)\right)$$

Normalizing the function,

$$C\left(\left(\frac{-2}{}\right)^2 + \left(\imath\frac{3}{}\right)^2 + \left(\imath\frac{3}{}\right)^2\right) = 1.$$

$$\left(\left(\sqrt{58}\right) \quad \left(\sqrt{58}\right) \quad \left(\sqrt{58}\right)\right)$$

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$$C = 58\left(\left(\frac{1}{4}\right) - \left(\iota\ 3\right)^{-2} + \left(\iota\ 3\right)^{-2}\right)$$

So,

$$\|\langle 1\ 1|\hat{L}_x|\Psi\rangle\|^2 =$$

$$\hat{L}_x|\Psi\rangle = \left(+\iota\ \frac{3}{\sqrt{29}}\hat{L}_x|1\ 0\rangle - \frac{4}{\sqrt{29}}\hat{L}_x|1\ -1\rangle\right)$$