This problem associated with chapter 7 was assigned during lecture.

Does $\hat{L}_{\mathcal{Y}}$ commute with $\hat{\mathbf{r}}^2$?

$$[\hat{L}_{\mathcal{Y}}, \hat{\mathbf{r}}^2] = \hat{L}_{\mathcal{Y}} \hat{\mathbf{r}}^2 - \hat{\mathbf{r}}^2 \hat{L}_{\mathcal{Y}}.$$

$$\hat{L}_{\mathcal{Y}} \hat{\mathbf{r}}^2 - \hat{\mathbf{r}}^2 \hat{L}_{\mathcal{Y}}.$$

Using the position representations, in spherical coordinates,

$$\hat{L}_{\mathcal{Y}} \doteq -\imath\hbar\partial/\partial\theta \text{ and } \hat{\mathbf{r}}^2 \doteq \mathbf{r}^2,$$

$$\hat{L}_{\mathcal{Y}} \hat{\mathbf{r}}^2 - \hat{\mathbf{r}}^2 \hat{L}_{\mathcal{Y}} = \mathbf{r}^2 \imath\hbar\partial/\partial\theta - \imath\hbar\partial/\partial\theta \mathbf{r}^2.$$

$\mathbf{r}^2$ has no θ dependence, so it can be separated from any quantity differentiated with respect to theta, I.E.,

$$\partial/\partial\theta \mathbf{r}^2 = \mathbf{r}^2 \partial/\partial\theta.$$

$$\mathbf{r}^2 \imath\hbar\partial/\partial\theta - \imath\hbar\partial/\partial\theta \mathbf{r}^2 = \mathbf{r}^2 \imath\hbar\partial/\partial\theta - \mathbf{r}^2 \imath\hbar\partial/\partial\theta = 0 = [\hat{L}_{\mathcal{Y}}, \hat{\mathbf{r}}^2] = 0.$$

$[\hat{L}_{\mathcal{Y}}, \hat{\mathbf{r}}^2] = 0$, so these quantities commute.