$$\hat{T}(a)|x\rangle = |x+a\rangle$$

$$|\Psi'\rangle = \hat{T}(a)|\Psi\rangle = \hat{T}(a) \int dx' |x'\rangle\langle x'|\Psi\rangle$$

$$= \int_{-\infty}^{\infty} dx' |x'+a\rangle\langle x'|\Psi\rangle$$

$$= \int_{-\infty}^{\infty} \delta(x - (x - a')) \Psi(x')$$

$$= \Psi(x-a) = \langle x|\Psi\rangle$$

$$\hat{T}(dx) = 1 - \imath/\hbar \, \hat{p}_x \, dx$$

$$\hat{T}(a) = \lim_{n\to\infty} [1 - \imath/\hbar \, p_x \, a/n]^n$$

$$= \exp(-\imath \, p_x \, a/\hbar)$$

$$[p] = [m \; L \; 1/T] \text{ (momentum)}$$

Commutation Properties
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$$(\hat{x} \, \hat{T}(\delta x) - \hat{T} \, \hat{x})|\Psi\rangle$$

$$= (\hat{x} \, \hat{T}(\delta x) - \hat{T}(\delta x) \, \hat{x}) \int_{-\infty}^{\infty} dx \, \langle x|\Psi\rangle|x\rangle$$

$$= \int_{-\infty}^{\infty} dx \, \hat{x} \, \hat{T}(\delta x)|x\rangle\langle x|\Psi\rangle - \int_{-\infty}^{\infty} dx \, \hat{T}(\delta x) \, \hat{x}|x\rangle\langle x|\Psi\rangle$$

$$= \int_{-\infty}^{\infty} dx \, \hat{x}|x+\delta x\rangle\langle x|\Psi\rangle - \int_{-\infty}^{\infty} dx \, \hat{T}(\delta x) \, \hat{x}|x\rangle\langle x|\Psi\rangle$$

$$= \int_{-\infty}^{\infty} dx \, (x + \delta x) \, \langle x|\Psi\rangle|x+\delta x\rangle - \int_{-\infty}^{\infty} dx \, x \, \langle x|\Psi\rangle|x+\delta x\rangle$$

$$= \delta x|\Psi\rangle$$

On the other hand...

$$\hat{x} \, \hat{T}(\delta x) - \hat{T}\backslash(\delta x) \, \hat{x} = \hat{x} - \imath/\hbar \, \delta x \, \hat{x} \, \hat{p}_x - \hat{x} + \imath/\hbar \, \delta x \, \hat{p}_x \, \hat{x}$$

$$= \imath/\hbar \, \delta x \, [\hat{p}_x, \hat{x}] = \delta x$$

$[\hat{p}_x, \hat{x}] = -\imath\hbar$

$[\hat{x}, \hat{p}_x] = \imath\hbar$ (The uncertainty principle)